9-1 day 1 Infinite Series

Learning Objectives:

I can find the sum of a geometric series (finite or infinite)

I can use a limit of a sequence of partial sums to find the sum of an infinite series

I can determine if a simple infinite series converges or diverges.

I can use the nth term test for divergence to determine if an infinite series diverges.

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$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....+\frac{1}{1024}$ is called a finite series and can also be expressed $\sum_{n=1}^{10}\frac{1}{2^n}$

When the series is an infinite series like

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....$

it can be written $\sum_{n=1}^{\infty}\frac{1}{2^n}$

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However, we cannot actually add up an infinite amount of numbers to find the sum so we'll need to go about this in a different way. We need to think of this as a **Sequence of Partial Sums**

$$S_1 = \frac{1}{2}$$

$$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$

$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$$

$$S_5 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{31}{32}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

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So the proper way to think about the sum of an infinite series is:

$$\lim_{n\to\infty}\sum_{k=1}^{n} a_n$$

So, in the previous example

$$\frac{1}{2},\frac{3}{4},\frac{7}{8},\frac{15}{16},\frac{31}{32},\frac{63}{64}....$$

We would think of this as a limit of the sequence of partial sums

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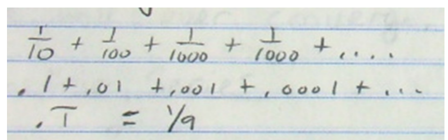
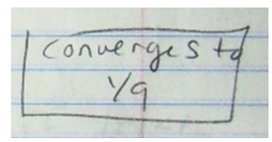
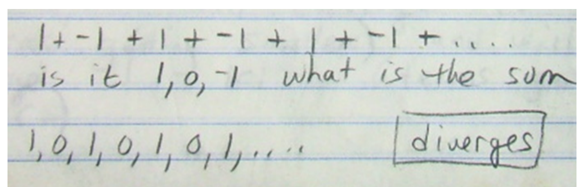
Ex1. Does each series converge or diverge? If it converges, identify what it converges to.



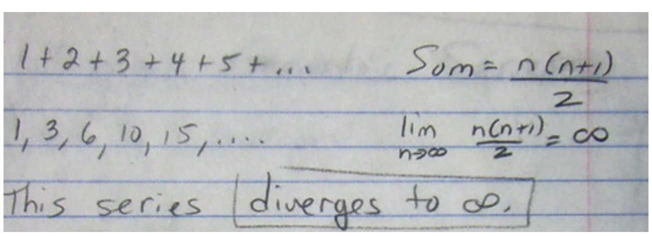


1.) $\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots$



2.) 1+-1+1+-1+1+. ..

3.) 1+2+3+4+5+….



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n^{th} Term Test for DIVERGENCE

If $\lim_{n\to\infty} a_n \neq 0$, then series diverges.

Please note that the converse is NOT TRUE:

If $\lim_{n\to\infty} a_n \neq 0$, the series converges. IS FALSE!

In reality, if $\lim_{n\to\infty} a_n \neq 0$ the series may converge or may diverge.

What the theorem is saying is that if the limit isn't 0, the series must diverge. It says nothing about converging.

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Arithmetic Series

Since $\lim_{n\to\infty} a_n \neq 0$ in an arithmetic series, we can conclude that NO infinite arithmetic series will ever converge. Hence, they aren't very interesting and we won't talk about them anymore.

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Geometric Series

$a_n = a_1 \cdot r^{n-1}$ will converge if $|r|<1$ (terms getting smaller) and will diverge if $|r| \geq 1$ (terms getting bigger)

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Infinite Geometric Series

$$\sum_{n=1}^{\infty} a_1 \cdot r^{n-1}$$

The infinite geometric series

converges to $S = \dfrac{a_1}{1-r}$

if |r| < 1.

Ex2. Decide if each infinite series converges or diverges. If it converges, find the sum.

1.) $\sum_{n=1}^{\infty} \frac{3}{2^n}$

2.) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

3.) $1+-\frac{1}{3}+\frac{1}{9}+-\frac{1}{27}+\frac{1}{81}+\ldots$

4.) $e+\frac{e^2}{3}+\frac{e^3}{9}+\frac{e^4}{27}+\ldots$

5.) $\sum_{n=1}^{\infty} 1-\frac{1}{n}$

6.) $\sum_{n=1}^{\infty} \left(\frac{1}{n}-\frac{1}{n+1}\right)$

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(1) $\sum_{n=1}^{\infty} \frac{3}{2^n} = \sum_{n=1}^{\infty} 3\left(\frac{1}{2^n}\right) = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \dots$

$r = 1/2$ so the series **Converge**

$S = \frac{a_1}{1-r} = \frac{3/2}{1-1/2} = \frac{3/2}{1/2} = \boxed{3}$

(2.) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \dots$

$r = 3/2$ so the series **diverges to ∞.**

(3.) $1 + -\frac{1}{3} + \frac{1}{9} + -\frac{1}{27} + \frac{1}{81} + \dots$

$r = -1/3$ so the series **converges**

$S = \frac{a_1}{1-r} = \frac{1}{1 - -1/3} = \frac{1}{4/3} = \boxed{3/4}$

(4) $e + \frac{e^2}{3} + \frac{e^3}{9} + \frac{e^4}{27} + \dots$

$r = \frac{e}{3} \approx .906$ so the series **converges**

$S = \frac{a_1}{1-r} = \frac{e}{1-e/3} = \frac{e}{\frac{3-e}{3}} = \frac{e}{1} \cdot \frac{3}{3-e} = \boxed{\frac{3e}{3-e}}$

(5.) $\sum_{n=1}^{\infty} 1 - \frac{1}{n}$ $\lim_{n\to\infty} 1 - \frac{1}{n} = 1$ **diverges**

Since $\lim_{n\to\infty} a_n \neq 0$, the series cannot converge.

Series diverges!

(6) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \lim_{m\to\infty} \sum_{n=1}^{m} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots \left(\frac{1}{m-1} - \frac{1}{m}\right) + \left(\frac{1}{m} - \frac{1}{m+1}\right)$

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$= 1 - \frac{1}{m}$ $\lim_{m\to\infty} 1 - \frac{1}{m} = 1$

this means that the series converges and its sum is $\underline{1}$.

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Homework

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